

Ukraine

An ideal quantum clock and Principle of maximum force

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We show that the space-time uncertainty relation for the quantum clock can be derived from the maximum force principle.

Achievement of required accuracy in any quantum measurement inevitably imposes certain limitations on characteristics of the device designed to perform it [1, 2]. Let us consider measurement of short time intervals. All possible methods to measure the time always involve observation of some physical process. The commonly known examples are periodic oscillations (all kinds of mechanical and electronic clocks), stationary flow of liquid (the clepsydra) or friable (the hourglass) substances, repeated motion of celestial bodies (the sundial and calendars). The so-called light-clock (a light signal periodically bouncing between two plane parallel mirrors, facing each other) plays a principal role in the Special Relativity. In the present paper we follow [3] to consider the so-called quantum clock based on observation of the radioactive decay described by the equation

$$\frac{dN}{dt} = -\lambda N, \quad (1)$$

where $N(t)$ is the current number of radioactive particles in the sample. Average number of the decayed particles (or nuclei) during the time interval $\Delta t \ll \lambda^{-1}$ is $\Delta N = \lambda N \Delta t$. It enables us to measure the time intervals calculating number of the decayed particles

$$\Delta t = \frac{\Delta N}{\lambda N}. \quad (2)$$

Relative error of such time measuring method is $\varepsilon = (\lambda N \Delta t)^{-1} = 1/\sqrt{\Delta N} \leq 1$. Increasing size of the quantum clock (the number N), one seemingly would gain unlimited improvement in accuracy of the time interval measurement. Burderi et al [3] showed however that it is not true.

Let us represent the measured time interval (2) in the following form

$$\Delta t = \frac{1}{\varepsilon^2 M c^2} \frac{E_p}{\lambda}, \quad (3)$$

where $M \equiv m_p N$ is mass of the clock, m_p is mass of the decaying particle and $E_p = m_p c^2$. It is desirable to get rid of the factor E_p/λ which characterizes the considered quantum clock.

Being a quantum device, the quantum clock must obey the standard uncertainty relation

$$\Delta E \cdot \Delta t \geq \hbar/2, \quad (4)$$

where ΔE is the maximum accuracy in energy of a quantum system achievable in the measurement process during the time interval Δt . Evidently, due to the energy conservation law, $\Delta E \leq E_p$ and $\Delta t \leq 1/\lambda$ because $\Delta N \leq N$ in (2). Thus $E_p/\lambda \geq \hbar/2$ and from (4) one obtains the lower limit of the quantum clock mass suitable to measure the time interval Δt with accuracy ε

$$M \geq \frac{\hbar}{2\varepsilon^2 c^2} \frac{1}{\Delta t}. \quad (5)$$

The result looks encouraging: sufficiently massive clock enables us to measure the short time intervals with the required accuracy. However some questions should be clarified.

In General relativity the time interval is a local characteristic, so in an inhomogeneous gravity field the quantum clock of finite size can measure only averaged (over the clock size) time intervals. One has to minimize the clock size in order to minimize the uncertainty in the time determination. Doing it, one inevitably faces the principal limitation. If the clock radius R (assuming spherical shape of the clock) becomes less than the corresponding gravitational radius $R_g = 2MG/c^2$, then we lose possibility to use the clock to measure time, as we cannot any more receive information about the decay products hidden from us behind the event horizon of the corresponding black hole that have formed. The condition $R > R_g$ can be rewritten in the form

$$\frac{1}{M} > \frac{2G}{c^2 R}. \quad (6)$$

Combining (6) with (5), one obtains

$$\Delta t R > \frac{1}{\varepsilon^2} \frac{G}{c^4} \hbar. \quad (7)$$

Treating R as uncertainty Δr in position of the physical object (the clock), which is the basis for the time measurement process, and taking into account that $\varepsilon \leq 1$,

one finally obtains [3]

$$\Delta t \Delta r > \frac{G}{c^4} \hbar. \quad (8)$$

The obtained inequality restricts the possibility to determine temporal and special coordinates of an event with arbitrary accuracy.

Any information about the particular process, which the clock is based on, is absent in the relation (8), so it suggests that this relation can be obtained from very general considerations. For that purpose we use the maximum force principle [4–6], which states that the tension or force between two bodies cannot exceed the value

$$F_{max} = \frac{c^4}{4G} \approx 3.25 \times 10^{43} N. \quad (9)$$

This limit does not depend on nature of the forces and is valid for gravitational, electromagnetic, nuclear or any other forces. It can be shown that the maximum force principle can be derived from the holographic principle and vice versa [7].

Existence of the maximum force is generally speaking a principle (postulate), but it is quite clear how it comes to being. Let us consider the attractive gravitational force between two masses M_1 and M_2 , separated by the distance D (we use the Newtonian approximation for simplicity). The forces between them are

$$F = G \frac{M_1 M_2}{D^2} = \left(\frac{GM_1}{c^2 D} \right) \left(\frac{GM_2}{c^2 D} \right) \frac{c^4}{G}. \quad (10)$$

As

$$M_1 M_2 \leq \frac{1}{4} (M_1 + M_2)^2,$$

then

$$F \leq \left[\frac{(M_1 + M_2)G}{c^2 D} \right]^2 \frac{c^4}{4G}. \quad (11)$$

Mutual approaching of the bodies is limited by the condition

$$R > R_g = \frac{2MG}{c^2},$$

which prohibits creation of the black hole with mass $M = M_1 + M_2$. Therefore

$$F \leq \frac{c^4}{4G}. \quad (12)$$

The surfaces which realize the maximum force (the maximum momentum flow) or the maximum power (the maximum energy flow) are horizons. A horizon appears at any attempt to surpass the force limit. And the horizon prohibits the possibility to surpass the limit.

Let us turn back now to the fundamental relation (8). Using the expression (9) for the limiting force, rewrite (8) in the form

$$\Delta t \Delta r > \frac{1}{F_{max}} \hbar. \quad (13)$$

With the Planck constant \hbar fixed, it is only the limiting force F_{max} which determines the restriction on the quantum clock size. If the force limiting value is absent in the theory, i.e. $F_{max} = \infty$, then $R_g \rightarrow 0$ and the limitation on the quantum clock size disappears.

In order to derive the relation (8) or (13), use the standard uncertainty relation

$$\Delta x_{min} \Delta p_{max} \geq \frac{\hbar}{2}. \quad (14)$$

Taking into account that

$$F_{max} = \frac{\Delta p_{max}}{\Delta x_{min}},$$

one immediately obtains that the minimum size of the clock, required to measure time intervals Δt_{min} , obeys the restriction

$$\Delta x_{min} \Delta t_{min} \geq \frac{\hbar}{F_{max}}, \quad (15)$$

which is identical to (13).

To conclude, we would like to stress that the main reason for the restriction on the quantum clock size is the requirement $R > R_g$, equivalent to the condition which prohibits formation of the horizon. Therefore it is quite natural that the righthand side of the relation (13) contains the limiting force value F_{max} , which can only be achieved on a horizon.

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